

viscosity : molecular picture

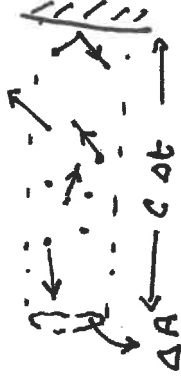
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• Viscosity of Gases :

Background : Kinetic theory of gases

n : number density of molecules

c : typical velocity of molecules



$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{(\text{momentum transferred in time } \Delta t)}{\Delta A \Delta t}$$

$$= \frac{1}{\Delta A \Delta t} \left(\text{No. of collisions in time } \Delta t \right) (\text{momentum transferred in each collision})$$

$$= \frac{1}{\Delta A \Delta t} \frac{1}{2} (n c \Delta t \Delta A) (2 m c)$$

$$= n m c^2$$

$$\propto n k_B T$$

$$\frac{1}{2} m c^2 = \frac{3}{2} k_B T$$

Equipartition theorem

force on
 ΔA in
 x dirⁿ

$$= \frac{\left(x\text{-momentum transferred} \right)}{\Delta t}$$

$$= \frac{1}{\Delta t} \left[\left(\begin{array}{c} \text{No. of mol.} \\ \text{from up} \end{array} \right) \cdot \left(\begin{array}{c} \text{typical} \\ x \\ \text{momentum} \end{array} \right) \right.$$

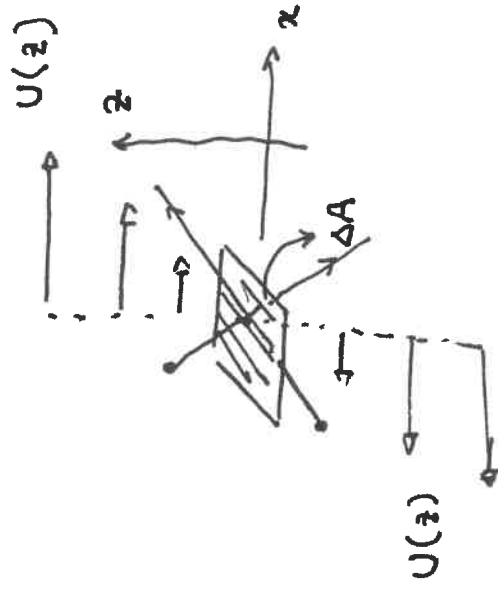
$$\left. - \left(\begin{array}{c} \text{No. of mol.} \\ \text{from down} \end{array} \right) \cdot \left(\begin{array}{c} \text{typical} \\ x \\ \text{momentum} \end{array} \right) \right]$$

$$= \frac{1}{\Delta t} \left\{ (n c \Delta A \Delta t) \cdot m [c + U(z)] - n c \Delta A \Delta t \cdot m [c + U(-z)] \right\}$$

$$= n c \Delta A \left[\cancel{m c} + m U(z) - \cancel{m c} - m U(z) \right]$$

$$= n c (\Delta A) m \left[U(0) + \frac{\partial U}{\partial z} \Delta z - U(0) - \frac{\partial U}{\partial z} \Delta z \right]$$

$$= m n c (\Delta A) \left(\frac{\partial U}{\partial z} \right) \Delta z$$



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$$\Rightarrow \sigma_{xz} = mnc(\Delta z) \left(\frac{\partial u}{\partial z} \right)$$

$\Delta z \sim$ mean free path $\equiv \lambda$

$$\Rightarrow \sigma_{xz} = \lambda mnc \frac{\partial u}{\partial z} \equiv \eta \frac{\partial u}{\partial z}$$

$$\Rightarrow \boxed{\eta = \lambda mnc}$$

$$mc \sim \sqrt{k_B T/m}$$

$\Rightarrow$

$$\eta \simeq mn\lambda \sqrt{k_B T/m}$$

$$\lambda \sim \frac{1}{na^2}$$

$$= \frac{m}{a^2} \sqrt{k_B T/m}$$

$\Rightarrow$

$$\boxed{\eta = \frac{m}{a^2} \sqrt{k_B T/m}}$$

- viscosity independent of density.
- Increases with temperature

Another form: mean free time $\tau = \frac{\lambda}{c}$

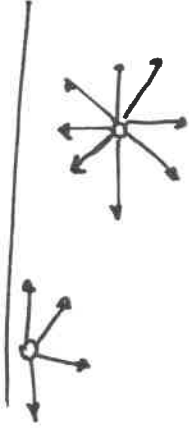
$\Rightarrow$

$$\boxed{\eta = \tau n k_B T}$$

Surface tension :

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Surface Physics is hard.



Molecules near the surface are missing some of its interaction.

Surface tension is the energy shortfall per-unit area.

$$\gamma \sim \frac{U}{2a^2}$$

$U \sim$ typical energy of interaction.

a : size of molecule

$$\text{oil : } U \sim k_B T \sim 4 \times 10^{-21} \text{ J}$$

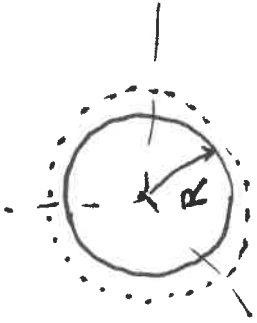
$$a \sim 10^{-9} \text{ m}$$

$$\Rightarrow \gamma \sim 4 \times 10^{-3} \frac{\text{J}}{\text{m}^2} \sim () \frac{\text{mN}}{\text{m}}$$

to In practice : oil : 20 mJ/m²

water 72 mJ/m²

5)



Energy

Energy argument: Area: $4\pi R^2 \rightarrow 4\pi(R+\Delta R)^2$

$$\Delta F = \gamma 4\pi(R+\Delta R)^2 - \gamma 4\pi R^2 - (\Delta p) 4\pi R^2 \Delta R$$

$$= \gamma 8\pi R \Delta R - \Delta p 4\pi R^2 \Delta R$$

$$= 0$$

$\Rightarrow$

$$\Delta p = \frac{2\gamma}{R}$$

Laplace pressure.

More generally:

$$\Delta p = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

Two radii of curvature

γ : Energy required to increase surface area by one unit.

Given a volume a liquid drop will try to minimize its surface area/energy

Oswald Ripening:



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The biggest drop grows bigger.

Growth of droplets:

$\Delta\mu$: difference in chemical potential between the two phases.

$$\left(\text{For a droplet of size } L \right) \Delta G \sim \Delta\mu \left(\frac{4}{3} \pi L^3 \right) - 4\pi R^2 \gamma L^2$$

$$\frac{\partial \Delta G}{\partial L} = 0 \Rightarrow (\Delta\mu) 4\pi L^2 - 4\pi \gamma 2L = 0$$

$$\Rightarrow L = \Delta\mu \sim \frac{2\gamma}{L}$$

The material that makes the droplet must be conserved

$$\Rightarrow \partial_t \phi + \text{div}(\vec{J}) = 0$$

The current is driven by gradient of $\mu \Rightarrow J = -r \vec{\nabla} \mu$

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$$\Rightarrow \partial_t \phi + \text{div}(-r \vec{\nabla} \mu) = 0$$

Integrate over a sphere of radius L

$$4 \cdot \frac{4}{3} \pi L^2 \frac{dL}{dt} + 4\pi L^2 (-r \vec{\nabla} \mu) = 0$$

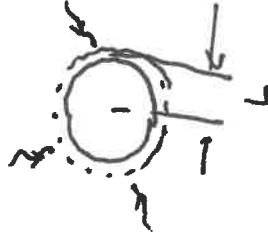
$$\Rightarrow \frac{4}{3} \frac{dL}{dt} \sim r \frac{2\gamma}{L^2}$$

$$\Rightarrow L^2 \frac{dL}{dt} \sim 2r\gamma$$

$$\boxed{L \sim t^{1/3}}$$

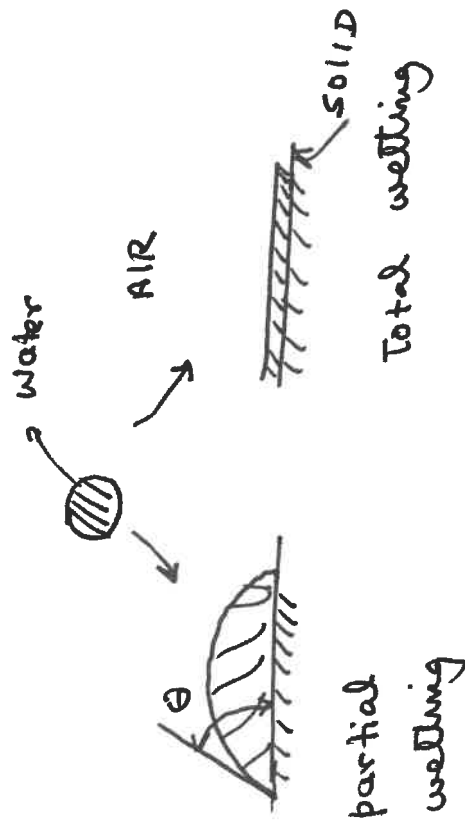
Lifshitz - Slyuzov - Wagner.

Applies to growth of droplets driven by diffusion.

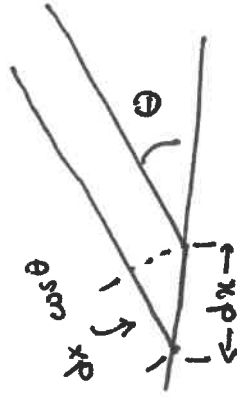


Contact with surfaces :

Wetting



$$S = \gamma_{SA} - (\gamma_{SL} + \gamma_{LA}) \quad \text{our old } \gamma$$



$$\delta F = (\gamma_{SA} - \gamma_{SL}) dx - \gamma \cos \theta dx = 0 \quad \text{at equilibrium}$$

$$\Rightarrow \gamma \cos \theta = \gamma_{SA} - \gamma_{SL}$$

or

$$S = \gamma (\cos \theta - 1)$$

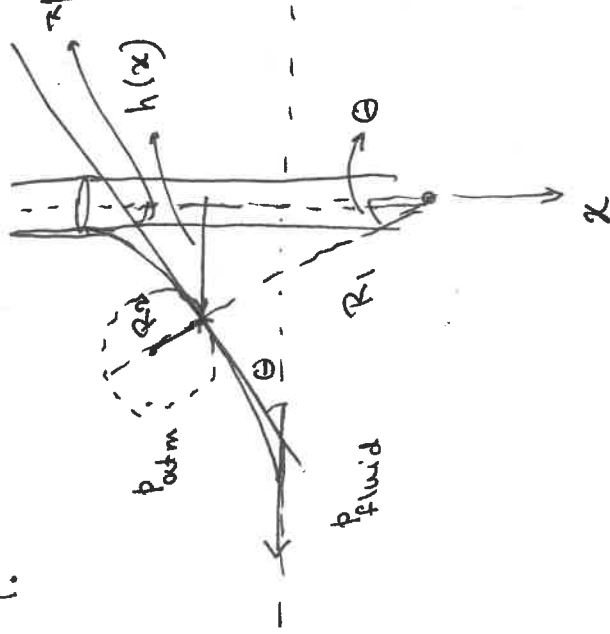
spreading
parameter

— Young-Dupré law

How to solve surface tension problems:

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1. $\pi/2 - \theta = \phi$



What is $h(x)$? (no gravity)

Laplace pressure:

$$\left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{\Delta p}{\gamma} = 0$$

Because p_{fluid} is uniform everywhere

$$\frac{dh}{dx} = h$$

$$\frac{d^2 h}{dx^2} = h$$

$$\frac{1}{R_1} = \frac{\cos \phi}{h} = \frac{1}{h(1+h^2)^{1/2}}$$

$$-\frac{1}{R_2} = -\frac{h}{(1+h^2)^{3/2}}$$

$$-\frac{h}{(1+h^2)^{3/2}} + \frac{1}{h(1+h^2)^{1/2}} = 0$$

$$\frac{1}{h} \frac{d}{dx} \left[\frac{h}{(1+h^2)^{1/2}} \right] = 0$$

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$$\Rightarrow \frac{h}{(1+h'^2)^{1/2}} = \text{constant} = B$$

Solution:

$$h = b \cosh\left(\frac{x}{b}\right)$$

catenary

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