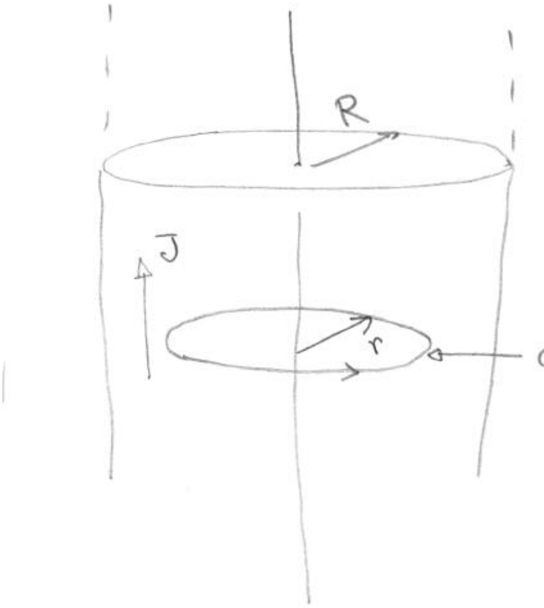
Solution to problem set $\overline{III}$

1. The application of Ampere's law in this problem is similar to the application of Gauss's law in problem set I.



$$\oint_{\Gamma} \vec{B} \cdot d\vec{l} = 2\pi r B(r)$$

For $r < R$ The current enclosed is

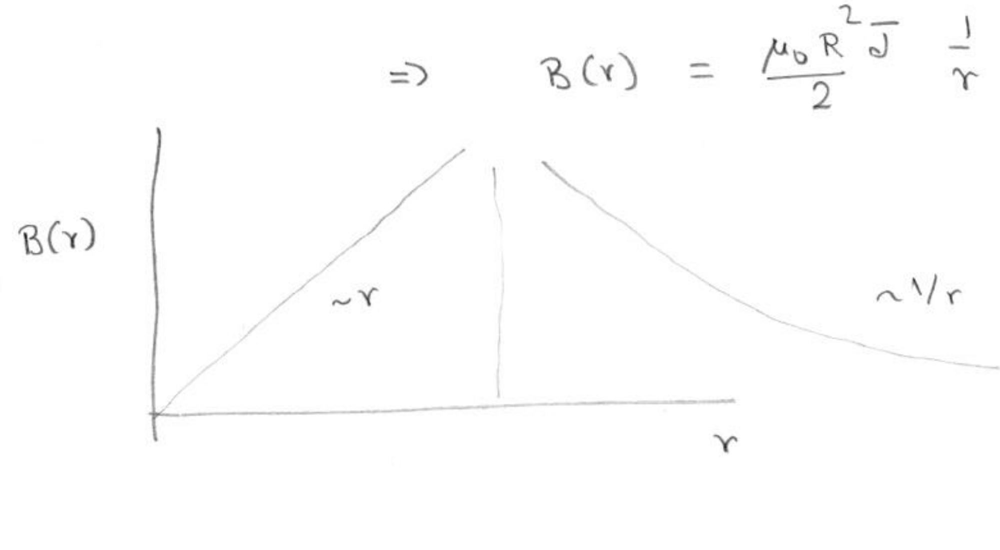
$$\pi r^2 J$$

For $r < R$, $2\cancel{\pi} r B(r) = \mu_0 \cancel{\pi} r^2 J$

$$\Rightarrow \quad B(r) = \frac{\mu_0 r}{2} J$$

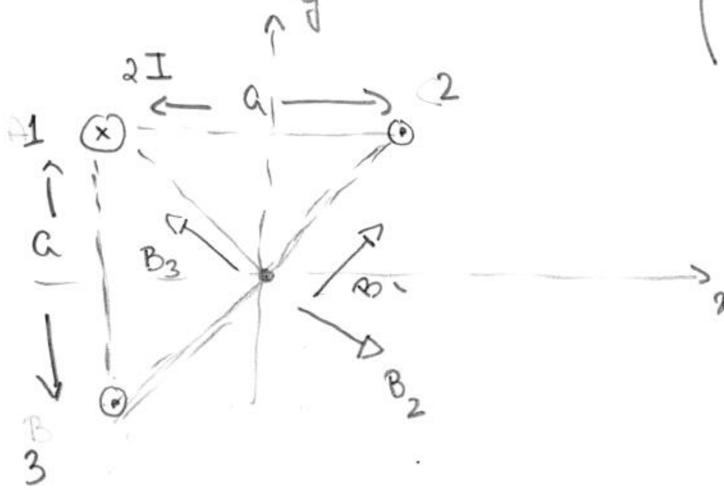
For $r > R$, $2\cancel{\pi} r B(r) = \mu_0 \cancel{\pi} R^2 J$

$$\Rightarrow \quad B(r) = \frac{\mu_0 R^2 J}{2} \frac{1}{r}$$



2. The magnetic field at a distance l from an infinitely long current carrying wire is

$$B = \frac{\mu_0 I}{2\pi l}$$



$$|\vec{B}_1| = \frac{\mu_0\, 2I}{2\pi\left(\frac{a^2}{4}+\frac{a^2}{4}\right)^{1/2}}$$

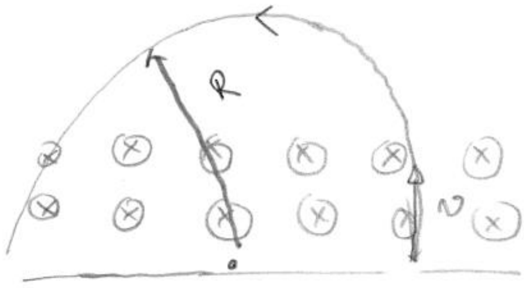
$$= \frac{\mu_0\, \cancel{2} I}{\cancel{2}\pi(a/\sqrt{2})}$$

$$= \frac{\sqrt{2}\,\mu_0}{\pi a}$$

$$\vec{B}_2 = -\vec{B}_3 = \frac{\mu_0}{\pi\sqrt{2}\,a}$$

The net magnetic field is a sum of these three field, $\vec{B}_2$ and $\vec{B}_3$ cancels each other, we are left with $\vec{B}_1$ whose magnitude is given above and direction shown in figure.

3.



The trajectory is a circle of radius R as sketched in figure

To obtain R,

$$q v B = \frac{m v^2}{R}$$

$$\Rightarrow \quad R = \frac{m v}{q B}$$

$$= \frac{10 \times 10^{-3}\ \text{kg} \cdot 844\ \text{m s}^{-1}}{1\ \text{coulomb} \cdot 1\ \text{tesla}}$$

$$= 8.44\ \text{m} \quad (\text{quite large})$$

4

$$\vec{B} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ -\frac{my}{r^3} & \frac{mx}{r^3} & 0 \end{vmatrix}$$

$$= \hat{x} \quad \partial_z\left(-\frac{mx}{r^3}\right) + \hat{y}\ \partial_z\left(-\frac{my}{r^3}\right)$$

$$+ \hat{z}\left[\partial_x\left(\frac{mx}{r^3}\right) + \partial_y\left(\frac{my}{r^3}\right)\right]$$

$$= +\hat{x}\ \frac{mx}{r^4}\, 3\, \frac{\partial r}{\partial z} + \hat{y}\ (-m)\ \frac{y\,(-3)}{r^4}\frac{\partial r}{\partial z}$$

$$+ \hat{z}\left[\frac{m}{r^3} - \frac{mx.3}{r^4}\frac{\partial r}{\partial x} + \frac{m}{r^3} - \frac{my\,3}{r^4}\frac{\partial r}{\partial y}\right]$$

$$r^2 = x^2 + y^2 + z^2$$

$$\Rightarrow \quad \frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

Substituting, we obtain

$$\vec{B} = \hat{x}\,\frac{3mx}{r^4}\,\frac{z}{r} + \hat{y}\,\frac{3my}{r^4}\,\frac{z}{r}$$

$$+ \hat{z}\left[\frac{m}{r^3} - \frac{3mx}{r^4}\,\frac{x}{r} + \frac{m}{r^3} - \frac{3my}{r^4}\,\frac{y}{r}\right]$$

$$= \hat{x}\,\frac{3mxz}{r^5} + \hat{y}\,\frac{3myz}{r^5}$$

$$+ \hat{z}\,\frac{m}{r^5}\left[r^2 - 3x^2 + r^2 - 3y^2\right]$$

$$= \hat{x}\,\frac{3mxz}{r^5} + \hat{y}\,\frac{3myz}{r^5}$$

$$+ \hat{z}\,\frac{m}{r^5}\left(3z^2 - r^2\right)$$

$$x^2 + y^2 = r^2 - z^2$$