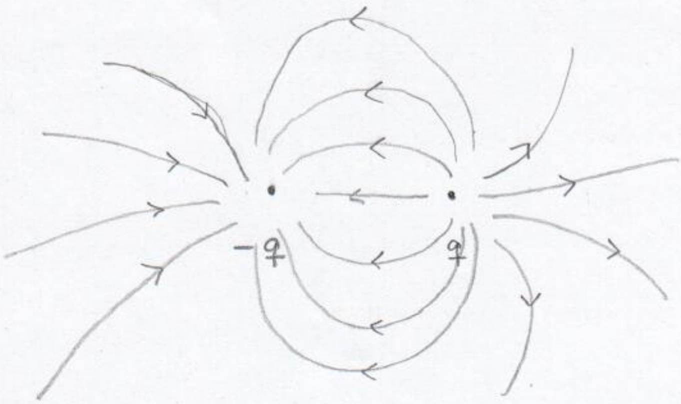
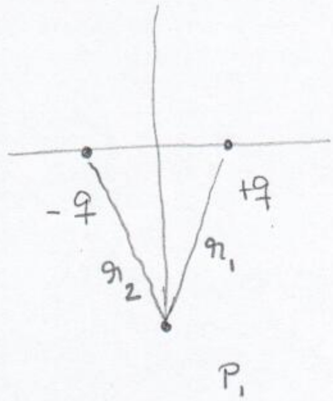
Solution to Problem set I

1.

(a)





(b) At the origin

$$\vec{E} = \left[-\hat{x}\,\frac{q}{d^2} \quad -\hat{x}\,\frac{q}{d^2} \right] \frac{1}{4\pi\epsilon_0}$$

$$= -\frac{1}{4\pi\epsilon_0}\,\frac{2q}{d^2}\,\hat{x}$$

(c) The work necessary is the potential at the point P_1

$$\Phi(P_1) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2} \right)$$

where $\quad r_1 = r_2 = \left(d^2 + h^2 \right)^{1/2}$

$$\Rightarrow \quad \Phi(P_1) = 0$$

The work done is zero.

(d)

Clearly, by symmetry

$$\phi(P_2) = \phi(P_3) = 0$$

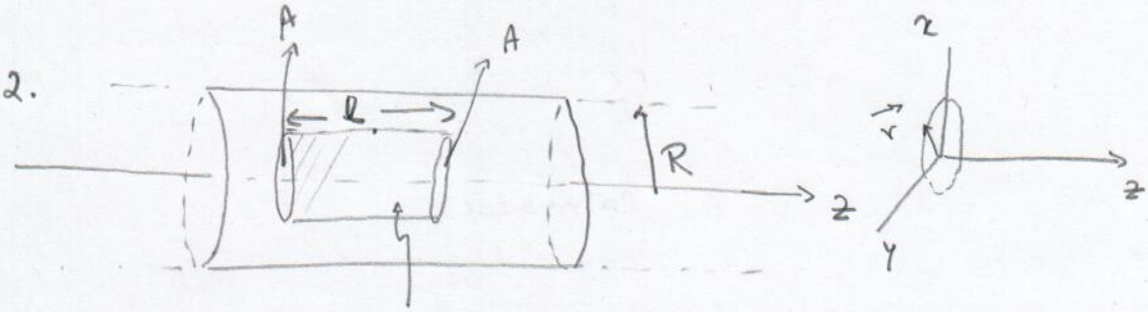
(e) As the potential at P_2 is equal to that at P_3 the work necessary is also zero.

Note that all points in the $y-z$ plane are equidistant from the two charges q and $-q$. Hence the potential of all those points are zero. The $y-z$ plane is an equipotential.

The plane extends to infinity, hence the work done to move any charge from infinity to this plane is also zero.

3.

2.



Gaussian surface.

By symmetry the electric field must be along $\hat{r}$ the radially outward direction as shown in figure. Also $\vec{E}$ is a function of r only. Using the Gaussian surface above, (the contribution from A and A' is zero)

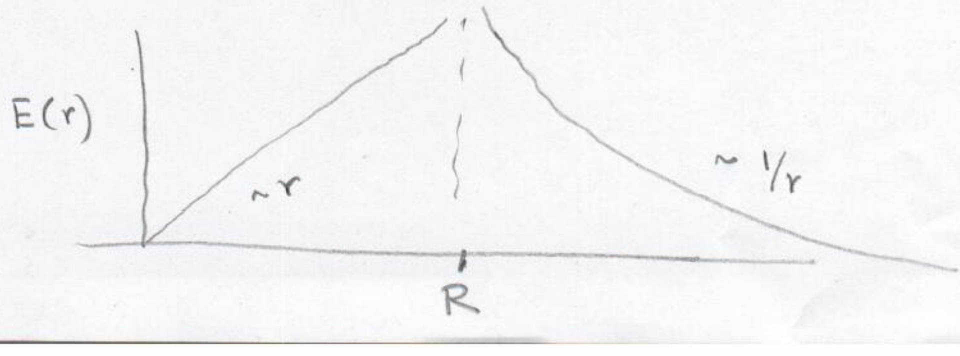
$$2\cancel{\pi} r \cancel{l}\, E(r) = \frac{1}{\epsilon_0} \cancel{\pi} r^2 \cancel{l} \rho$$

$$\Rightarrow \qquad E(r) = \frac{1}{\epsilon_0} \frac{r\rho}{2} \qquad \text{for} \quad r < R$$

For $r > 0$, the total charge enclose is $\pi R^2 l \rho$, so we obtain

$$2\cancel{\pi} r \cancel{l}\, E(r) = \frac{1}{\epsilon_0} \cancel{\pi} R^2 \rho \cancel{l}$$

$$\Rightarrow \qquad E(r) = \frac{1}{\epsilon_0}\left(\frac{\rho R^2}{2}\right)\frac{1}{r}$$



3. The potential $\phi = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \leftarrow$ total charge

surface charge density

$$\sigma = \frac{Q}{4\pi R^2} = \frac{1}{\cancel{4\pi} R^{\cancel{2}}}\cancel{4\pi}\epsilon_0 \cancel{R}\,\phi$$

$$\sigma = \frac{\epsilon_0 \phi}{R}$$

No. of extra electrons

$$N = \frac{\sigma}{\underset{\substack{\uparrow\\ \text{electronic}\\ \text{charge}}}{e}} = 4\pi\epsilon_0 \frac{\phi}{4\pi R}\frac{1}{e}$$

$$= \frac{1}{9\times 10^9}\frac{10^3 \times \cancel{2\pi}}{\underset{2}{\cancel{4\pi}} \times 7.5 \times 10} \times \frac{1}{1.6\times 10^{-19}}\ \frac{C}{m^2}$$

For a basketball $2\pi R = 29.5$ inches

$\simeq 30$ inches.

$= 30 \times 2.54$ cm

$\Rightarrow R = \frac{7.5\times 10}{2\pi}$ cm

$$= \frac{10^3}{10^{-9}}\frac{1}{9\times 2\times 7.5}\ C\,m^{-2}$$

$$\simeq 10^{12}$$