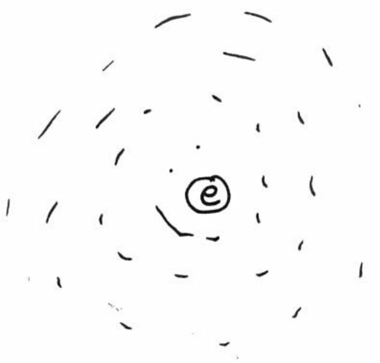
1.



The total negative charge:

$$Q = \int_0^\infty \rho(r)\, 4\pi r^2\, dr$$

$$= -c\int_0^\infty e^{-2r/a}\, 4\pi r^2\, dr \qquad \frac{r}{a} = \xi$$

$$= -c\,4\pi\int_0^\infty e^{-2\xi}\, a^2\xi^2\, a\, d\xi$$

$$= -c\,4\pi a^3\int_0^\infty \xi^2 e^{-2\xi}\, d\xi$$

$$= -4\pi a^3 c\left[\xi^2\frac{e^{-2\xi}}{-2}\Big|_0^\infty + \int_0^\infty \cancel{2}\xi\,\frac{e^{-2\xi}}{+\cancel{2}}\, d\xi\right]$$

$$= -\frac{4\pi a^3 C}{2}\left[\cancel{2}\xi\,\frac{e^{-2\xi}}{-2}\Big|_0^\infty + \int_0^\infty \frac{e^{-2\xi}}{+2}\, d\xi\right]$$

$$= -4\pi a^3 c\left[\frac{1}{2}\,\frac{e^{-2\xi}}{-2}\Big|_0^\infty\right]$$

$$= +\frac{4\pi a^3 C}{4}\left[0 - 1\right] = -\pi a^3 C$$

As total charge must be zero, $Q = -e$

$$\Rightarrow \quad \cancel{c = \frac{e}{4a^3}} \qquad \boxed{C = \frac{e}{\pi a^3}}$$

(b) The total electric charge inside a volume of radius R is

$$Q_{enc}(R) = \int_0^R \rho(r)\, 4\pi r^2\, dr + e$$

Let us evaluate the integral:

$$\int_0^R \rho(r)\, 4\pi r^2\, dr = -\frac{e}{\pi a^3} 4\pi \int_0^R e^{-2r/a} r^2\, dr$$

$$= -\frac{e \cdot 4\cancel{\pi}}{\cancel{\pi} a^3} \int_0^{R/a} e^{-2\xi} a^2 \xi^2\, a\, d\xi \qquad \frac{r}{a} = \xi$$

$$= -\frac{e\, 4 a^3}{a^3} \int_0^{R/a} e^{-2\xi} \xi^2\, d\xi$$

$$= -e4 \left[\xi^2 \frac{e^{-2\xi}}{-2} \Bigg|_0^{R/a} + \int_0^{R/a} \cancel{2}\xi \frac{e^{-2\xi}}{+\cancel{2}}\, d\xi \right]$$

$$= -e4 \left[\left(\frac{R}{a}\right)^2 \frac{e^{-2R/a}}{-2} + \xi \frac{e^{-2\xi}}{-2} \Bigg|_0^{R/a} + \int_0^{R/a} \frac{e^{-2\xi}}{+2}\, d\xi \right]$$

$$= -e4 \left[-\frac{1}{2} \frac{R^2}{a^2} e^{-2R/a} + \left(\frac{R}{a}\right) \frac{e^{-2R/a}}{-2} + \frac{1}{2} \frac{e^{-2\xi}}{-2} \Bigg|_0^{R/a} \right]$$

$$= -4e \left[-\frac{R^2}{2a^2} e^{-2R/a} - \frac{R}{2a} e^{-2R/a} + \frac{e^{-2R/a}}{-4} + \frac{1}{4} \right]$$

$$= -4e \left[e^{-2R/a} \left\{ -\frac{1}{2}\frac{R^2}{a^2} - \frac{1}{2}\frac{R}{a} - \frac{1}{4} \right\} + \frac{1}{4} \right]$$

$$\Rightarrow \int_0^R \rho(r)\, 4\pi r^2\, dr = -e\left[-e^{-2R/a}\left(\frac{2R^2}{a^2} + \frac{2R}{a} + 1\right) - 1\right]$$

The total enclosed charge

$$\cancel{Q_{enc} = e^{-2}}$$

$$Q_{enc} = q_e\, e^{-2R/a}\left[1 + \frac{2R}{a} + \frac{2R^2}{a^2}\right]$$

$\downarrow$

where we changed notation and called q_e the electronic charge.

The total charge inside a volume of radius 'a' is

$$Q_{enc}(a) = q_e\, e^{-2}\left[1 + 4\right]$$

$$\Rightarrow \boxed{Q_{enc}(a) = q_e\, 5 e^{-2}}$$

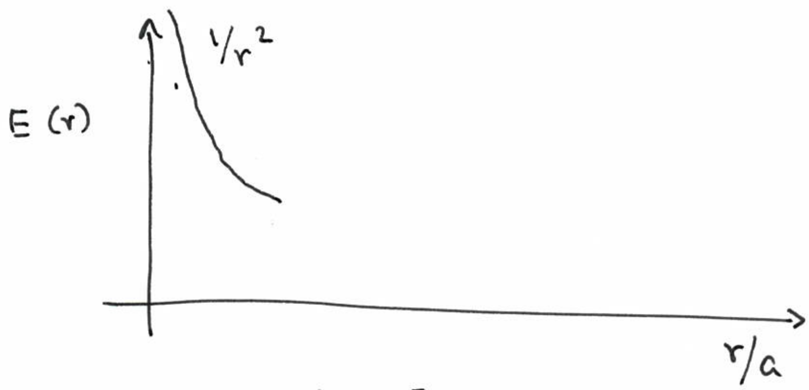
(c) By symmetry the electric field will depend only on the radial coordinate and also have only radial component.

$$\Rightarrow \vec{E} = E(r)\,\hat{r}$$

Applying Gauss' law to a surface of a sphere of radius r, we have:

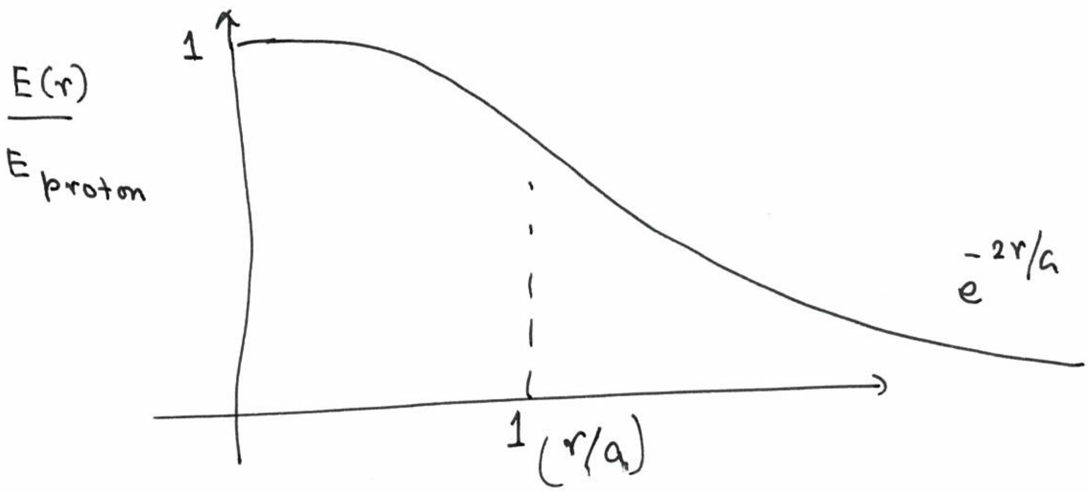
$$E(r)\, 4\pi r^2 = \frac{1}{\epsilon_0} Q_{enc}(r)$$

$$\Rightarrow \quad E(r) = \frac{1}{4\pi\epsilon_0} \frac{q_e}{r^2} \left[1 + \frac{2r}{a} + \frac{2r^2}{a^2} \right] e^{-2r/a}$$



as $r \to 0$, $\quad E(r) = \frac{q_e}{4\pi\epsilon_0} \left[\frac{1}{r^2} + \frac{2}{ar} + \frac{2}{a^2} \right] + \dots$

It is better to plot

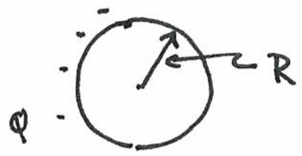


$$\cancel{\underline{E}(r) = \frac{\varepsilon}{4\pi\epsilon_0 r^2}\left[1 + \left(\frac{5}{2e} - 1\right)\left(\frac{r}{a}\right)^3\right]}$$

(d) Zero dipole moment.

2.

(a)



$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = -0{\cdot}15 \text{ volt}$$

$$Q = (4\pi\epsilon_0)\, R\,(-0{\cdot}15) \text{ volt}$$

$$= (4\pi\epsilon_0)\, 3\times 10^{-7}\,(-0{\cdot}15) \text{ C}$$

$$N = \frac{Q}{\varepsilon}$$

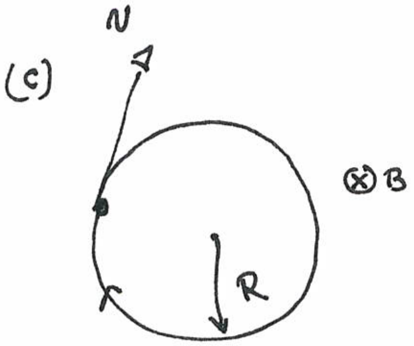
$\varepsilon \leftarrow$ electronic charge.

3 (b)

$$E(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2} = \frac{V}{R}$$

(c)

(c)



$$F = q\,(v \times B)$$

$$\frac{mv^2}{R} = qvB$$

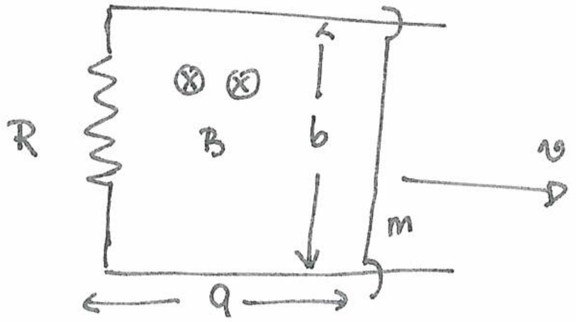
$$\Rightarrow \quad R = \frac{mv^2}{qBv} = \frac{mv}{qB}$$

(d)

$$T = \frac{2\pi R}{v} = 2\pi \frac{m\cancel{v}}{qB}\frac{1}{\cancel{v}}$$

$$= \frac{2\pi m}{qB}$$

3.



(a) Total flux $\Phi = abB$ $\qquad v = \frac{da}{dt}$

rate of change of flux $\frac{d\Phi}{dt} = bBv$

This will induce an emf $\mathcal{E} = -\frac{d\Phi}{dt}$

argument I : Lenz's law implies that the induced emf will stop the change of flux. so the bar should slow down.

argument II A current will be set up. The current passing through resistance R will loose energy. Hence motion should stop.

(b) The current $I = \frac{\mathcal{E}}{R}$

$$= \frac{bBv}{R}$$

The eqn for the metal bar :

$$m \frac{dv}{dt} = \text{force}$$

$$= q\, v\, B$$

total charge

$$= q\, b.\, v\, B$$

charge per unit length.

What is the initial kinetic energy?

$$\frac{1}{2} m v^2$$

Initially : $\mathcal{E}$ = total energy $= \frac{1}{2} m v^2$

Rate of change of energy:

$$\frac{d\mathcal{E}}{dt} = \frac{\cancel{1}}{\cancel{2}} m \cancel{2} v \frac{dv}{dt}$$

$$= m v \frac{dv}{dt}$$

$$\left(\begin{array}{c} \text{Rate of} \\ \text{energy dissipation} \\ \text{in the resistor} \end{array} \right) = I^2 R \qquad *$$

$$m v \frac{dv}{dt} = I^2 R = \frac{b^2 B^2}{R^{\cancel{2}}} v \cancel{R}$$

$$m \cancel{v} \frac{dv}{dt} = \frac{b^2 B^2}{R} \cancel{v}$$

$$\frac{dv}{dt} = \frac{b^2 B^2}{R m}$$

$$v_f - v_i = \int_0^T \frac{b^2 B^2}{mR}\, dt$$

$$0 - v^{\circ} = \frac{b^2 B^2}{mR} T$$

$$\boxed{T = \frac{m v R}{b^2 B^2}}$$

Moving with constant acceleration:

distance ~~coverg~~ covered

$$s = vt + \frac{1}{2}(\text{acceleration})\, t^2$$

$$= vT - \frac{1}{2}\left(\frac{b^2 B^2}{Rm}\right) T^2$$

(c) Energy* is not being conserved but dissipated in the resistor.

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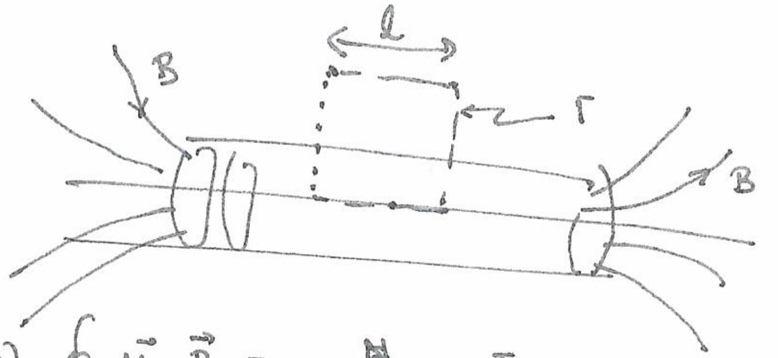
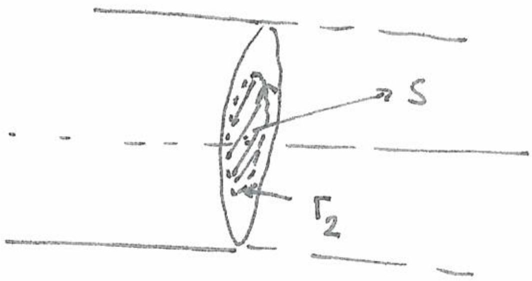
4



(a) $\oint_{\Gamma} d\vec{l} \cdot \vec{B} = \mu_0 I_{enc}$

$B l = \mu_0 N l I$

$B = \mu_0 N I$



(b) $B = 0{\cdot}4$ Tesla

$I = 10$ Amp.

$N = \frac{B}{\mu_0 I} = \frac{1}{\text{meter}}$

(c) $\oint_{\Gamma_2} \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$ $\quad\Phi$: flux through S

$2\pi r E = -\frac{d}{dt} B \pi r^2$

$B = B_0 \cos \omega t$

$= -\pi r^2 \frac{dB}{dt}$

$\frac{dB}{dt} = B_0 \omega \sin \omega t$

$= -\pi r^2 B_0 \omega \sin \omega t$

$2\cancel{\pi}\cancel{r} E = \pi r^{\cancel{2}} \omega B_0$, $\boxed{E = \frac{\cancel{\pi} r \omega B_0}{2\cancel{\pi}} = \frac{\omega r B_0}{2}}$

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$$E = \hat{z}\, E_0 \sin(y - vt)$$

$$B = \hat{x}\, B_0 \sin(y - vt)$$

The Maxwell's eqns in free-space: $\nabla \cdot E = 0$

$\nabla \cdot B = 0$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t}$$

clearly, with the give E and B, ~~$\nabla \cdot E_0$~~

$\nabla \cdot E = 0$, and $\nabla \cdot B = 0$.

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & E_0 \sin(y - vt) \end{vmatrix}$$

$$= \hat{x}\, E_0 \cos(y - vt)$$

$$\nabla \times B = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ B_0 \sin(y - vt) & 0 & 0 \end{vmatrix}$$

$$= \cancel{\hat{z}\, B_0 \sin(y - vt)} = -\hat{z}\, B_0 \cos(y - vt)$$

$$\frac{\partial E}{\partial t} = +v\, \hat{z}\, E_0 \cancel{\sin} \cdot \cos(y - vt)$$

To satisfy $\nabla \times B = \frac{1}{c^2}\frac{\partial E}{\partial t}$, we have $B_0 = +\frac{v E_0}{c^2}$

To satisfy $\nabla \times E = -\frac{\partial B}{\partial t}$, $\frac{\partial B}{\partial t} = -v\,\hat{x}\,B_0 \cos(y - v$

$\Rightarrow \quad E_0 = +\, v B_0$

One solution is $\boxed{v = c \,, \quad E_0 = c B_0}$

(b) A wave propagating in the $-x$ direction has the equation:

~~Sin x + c~~

$\sin(kx + \omega t)$ where

$c = \frac{\omega}{k}$ is the speed of light.

$\omega = 2\pi f = 2\pi \times 100 \times 10^6$ Hz

As the wave is propagating along the x direction ~~and~~ ~~E~~ then $\vec{E}$ must be in the $y-z$ plane. E is perpendicular to $\hat{z}$ so it must be along $\hat{y}$

$\vec{E} = \hat{y}\, E_0 \sin(kx + \omega t)$ $\quad k = \omega c$

$\Rightarrow \quad \vec{B} = \hat{z}\, B_0 \sin(kx + \omega t)$