Electromagnetism (FK4005e)
Final Examination

Each of the following problems carry 20 marks. Solve any 5 of the 6 problems given below.

1. A neutral hydrogen atom in its normal state is sometimes modeled as an electric charge distribution of the following type:
   A point charge of $+e$ at the center surrounded by a cloud of negative charge which has the following distribution of negative charge density

$$\rho(r) = -C\exp(\frac{-2r}{a}) \tag{1}$$

   where $a = 0.53 \times 10^{-10}$m is called the Bohr radius, e is the magnitude of the electronic charge, and C is a constant.

   (a) Clearly the value of C should be such that the atom is charge-neutral. This implies that the total negative charge in the cloud should be equal to $-e$. Find C. (5p)

   (b) What is the total electric (sum of positive and negative charges) charge inside a volume of radius a ?(5p)
   Hint: As ρ is a function of r only the amount of negative charge in a spherical shell between $r = R_1$ to $r = R_2$ is given by

$$\int_{R_1}^{R_2} \rho(r) 4\pi r^2 dr \tag{2}$$

   (c) Calculate the electric field due to this atom everywhere, for $r < a$ and $r > a$. Make a rough plot of the electric field as a function of r. (9p)
   Hint: Use Gauss's law to calculate the electric field due to the negatively charged cloud. Then add to it the field due to the positive (point) charge at the center to get the total electric field. You do not need to calculate the electric field very close to $r = 0$.

   (d) What is the dipole moment of this atom ? (1p)

2. An interstellar dust grain, roughly spherical in shape with radius $R = 3 \times 10^{-7}$m, has acquired a negative charge such that its potential is $V = -0.15$volt.

   (a) How many extra electrons have it picked up ? (5p)

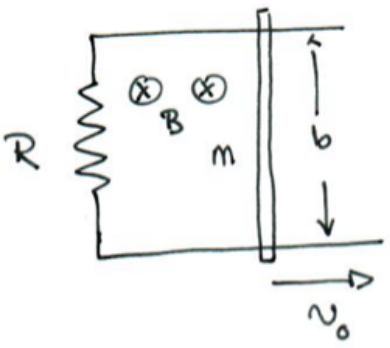
   (b) What is the strength of the electric field on its surface ? (5p)

   (c) Assume that the mass of the dust grain is $m = 10^{-13}$gm. Suppose it is moving with a speed v (which is very small compared to c, the speed of light) in a plane perpendicular to the interstellar magnetic field $B = 3 \times 10^{-6}$gauss. Make a sketch showing the circular path the dust grain takes, the magnetic field and the velocity v. What is the radius, R of the circular path ? (5p)

   (d) How many years does it take to complete one orbit ? (5p)
   Hint: The time to complete one orbit will be independent of v, which is why it is not given.

3. A metal crossbar of mass m slides without friction on two long parallel conducting rails a distance b apart. A resistor R is connected across rails at one end; compared with R the resistance of the bar and the rails are negligible. There is a uniform field $\boldsymbol{B}$ perpendicular to the plane of the figure. At time $t = 0$ the crossbar is given a velocity v_0 towards the right. What happens then ? (Ignore mechanical friction between the bar and the rails).

   (a) Does the rod ever stop moving ? If so, when ? (8p)



(b) How far does it go ? (8p)

(c) How about conservation of energy ? (4p)

4. Consider a long straight solenoid which carries a steady current I with number of turns N per unit length.

   (a) If I is kept steady, what is the magnetic field at a point on the axis of this solenoid, that is far from the ends of the solenoid ? You can do this using Ampere's law. State clearly what are the assumptions you need to make. (5p)

   (b) A solenoid designed for whole-body imaging (MRI) may have a field as large as 0.4 Tesla at its center. Assuming $I = 10$ amperes calculate N ? (5p)

   (c) The current in a solenoid is driven by a radio-frequency power source. The magnetic field oscillates at 2.5×10^6 cycles per second with an amplitude of 4 gauss. What is the amplitude of the oscillating electric field at a point 3cm from the axis. (The radius of the solenoid a larger than 3cm.) (10p)

5. Start with the source-free (no charge and no current) Maxwell's equations in free space. If the following expressions for electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$

$$\boldsymbol{E} = \hat{z} E_0 \sin(y - vt) \tag{3}$$
$$\boldsymbol{B} = \hat{x} B_0 \sin(y - vt) \tag{4}$$

   are solutions of the (source-free) Maxwell's equations then :

   (a) What conditions must E_0, B_0 and v must satisfy ? (10p)

   (b) A plane electromagnetic sinusoidal wave is propagating along the $-\hat{x}$ direction. Its frequency is 100megahertz, the electric field is perpendicular to the $\hat{z}$ direction. Write down the equations that the electric and magnetic field satisfies. (10p)
   Hint: Remember that the wavelength and the frequency of the wave are related through the speed of light.

6. Circuit theory

   (a) What is (always) the sum of the currents arriving at a node? Why cannot all the currents have the same sign/direction? (2p)

   (b) Suppose you want to split a voltage V into two voltages V_1 and V_2 such that you get another (lower) voltage output. Draw a circuit diagram and explain how the three voltages are related. Assume the input voltage is $V = 12\,\mathrm{V}$ and the ratio between the two resistors is $R_1/R_2 = 3/5$. Determine the two corresponding voltages V_1 and V_2. (4p)

   (c) Formulate Ohm's law in an AC setting. Explain why $V_0 = R\,I_0$ is not generally true, where V_0, I_0 are the voltage and current amplitudes, respectively, and R is the resistance. Under which condition does $V_0 = R\,I_0$ hold? (4p)

(d) How does a resistor, a capacitor and an inductor change the phase between the voltage and the current in an AC circuit? Motivate your answers. (4p)

(e) Consider a resistor ($R = 2000\,\Omega$), a capacitor ($C = 1.0\,\mu$F) and an ideal coil ($L = 20\,$mH) coupled in series and connected to an AC generator producing a 15 V output. Write down an expression for the impedance (or draw the impedance plane and indicate all relevant components) and determine the resonance frequency ω_0 of this circuit. Also calculate the numerical value of the Q factor of this circuit. You can use the expression for Q factor from the physics handbook. (6p)