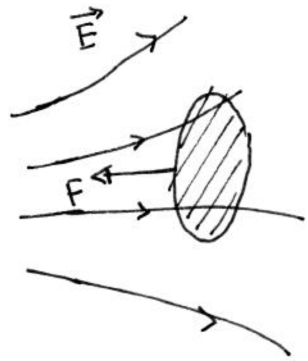
Lecture 14

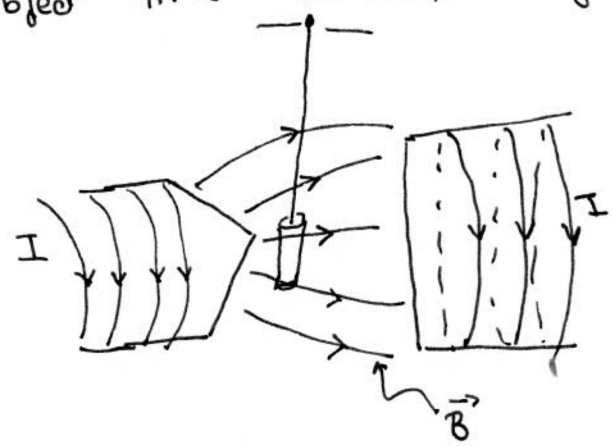
Magnetic materials

14.1 We have shown that if you introduce a dielectric in an inhomogeneous magnetic field, it is attracted towards the part of the field where ~~it~~ ~~is~~ the field is stronger. This is because in an electric field dipoles are induced. The dipole are attracted towards the part of the field where the field is stronger, because the force on a dipole is

$$\vec{F} = \left(\vec{p} \cdot \vec{\nabla}\right) \vec{E}$$



what happens if I indrodue an object in a non-unifor magnetic field ?



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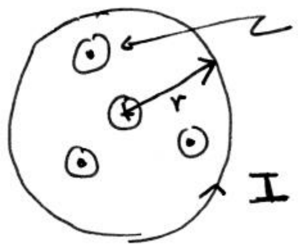
Naively what we expect is the following:

~~B~~ The material contains atoms. In atoms electrons are moving about a nucleus. This creates a current loop. This is not a constant current but a fluctuating one (at the time scale of rotation of the electron). Fluctuating current sets up a fluctuating magnetic field. Fluctuating magnetic field sets up a fluctuating electric field; and so on to an electromagnetic wave. This wave carries away energy so the electron slows down and eventually collapses onto the nucleus. But this catastrophy is cured if the problem is treated quantum mechanically. So, in principle, we cannot deal with this problem unless we know quantum mechanics. But for the moment ignore this (very valid) objections and assume that we treat the atomic currents as steady currents.

Then we have the situation ~~that~~ :



As we turn on the external magnetic field, the flux enclosed by the atomic current changes.

By Faraday's law this changing flux sets up an electric field. This electric field should decrease the atomic current, because of Lenz's law.

$$2\pi r E = -\frac{d}{dt}\left(\pi r^2 B\right)$$

$$\Rightarrow \quad E = -\frac{r}{2}\frac{dB}{dt}$$

This electric field produces a torque

$$\Gamma = q r E = -\frac{r^2}{2}\frac{dB}{dt} q$$

By Newton's law, the rate-of-change of angular momentum of the electron is

$$\frac{dJ}{dt} = \Gamma$$

$\Rightarrow$ The total change of angular momentum, when B is changed from zero to B_{max} is,

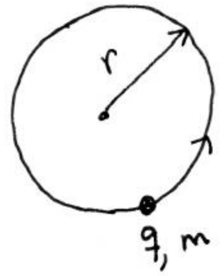
$$\Delta J = \int_0^T \Gamma \, dt$$

$$= -\frac{r^2}{2} q \int_0^T \frac{dB}{dt} \, dt$$

$$= -\frac{qr^2}{2} \int_0^{B_{max}} dB$$

$$\Delta J = -\frac{qr^2}{2} B$$

For a charged particle of mass m and charge q moving in a circular orbit we have



angular momentum $J = r m v$

magnetic moment $\mu = I \pi r^2$

$$= q \frac{v}{2\pi r} \pi r^2$$

$$= \frac{q v r}{2}$$

$$\Rightarrow \quad = \boxed{\vec{\mu} = \frac{q}{2m} \vec{J}}$$

For electron, of course, q is negative, so

$$\vec{\mu} = -\frac{|q|}{2m}\vec{J}$$

Miraculously this is a relationship true for orbital electrons even in quantum mechanics.

$$\Delta\mu = +\frac{q_e}{2m}\Delta J$$

$$= +\frac{q_e}{2m}\left(-\frac{q_e r^2}{2}\right)B$$

$$\boxed{\Delta\mu = -\frac{q^2 r^2}{4m}B}$$

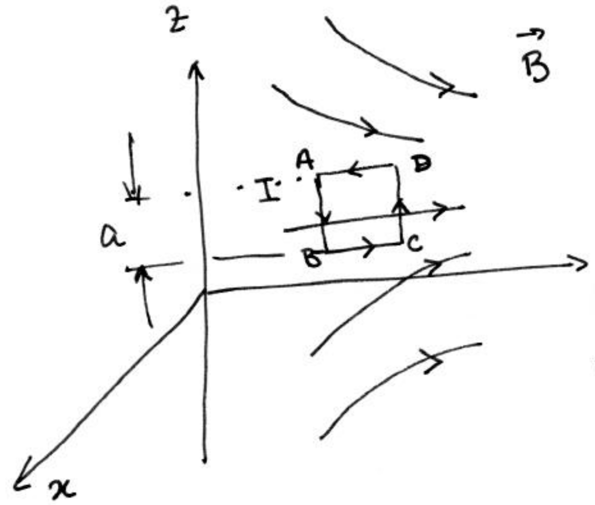
This sign is always negative irrespective of whether the current is set up by moving +ve ~~one~~ or -ve ~~hen~~ charges.

So by turning on the magnetic field we shall change the magnetic moment in the molecules of the material. And this change is proportional to the magnetic field.

Dielectric	Naive magnetic material
1. External electric field sets up molecular electric dipole moments.	1. Ext. magnetic field <u>changes</u> molecular magnetic moments.
2. Induced dipole moment $p = \alpha E_{ext}$ (α: polarizability of the atom)	2. Change in magnetic moment $\Delta\mu = -\frac{q^2 r^2}{4m} B$
3. Electric dipoles are attracted towards stronger E field	3 ? ?

14.2 ~~...~~ Magnetic moments in external magnetic field

Remember from Example 6·1 that the force on a current loop in an external magnetic field is zero (If the magnetic field is constant in space). The same as an electric dipole in an uniform electric field. But what happens if it is non-uniform?



B_y increases with y.

~~F_{AB} =~~

[Remember, net force $\vec{F} = I\oint_{\Gamma} d\vec{l} \times \vec{B}$]

$$F_{AB} = \hat{x}\, a\, I\, B(y)$$

$$F_{BC} = 0, \qquad F_{DA} = 0$$

$$F_{CD} = -\hat{x}\, a\, I\, B(y+a)$$

$$F = F_{AB} + F_{BC} + F_{DA} + F_{CD}$$

$$= \hat{x}\, a\, I \left[B_y(y) - B_y(y) - \frac{\partial B}{\partial y} a + O(a^2) \right]$$

$$= -\hat{x}\, I a^2 \frac{\partial B_y}{\partial y}$$

⇒ ~~more generally~~

~~$\vec{F} = \mu$~~

Note that $\dfrac{\partial B_x}{\partial x} + \dfrac{\partial B_y}{\partial y} + \dfrac{\partial B_z}{\partial z} = 0.$

~~As~~ we also assumed $\partial B_z/\partial z = 0$ $[F_{BC} = F_{DA} = 0]$

$$\Rightarrow \quad \frac{\partial B_x}{\partial x} = -\frac{\partial B_y}{\partial y}$$

$$\Rightarrow \quad \vec{F} = \hat{x}\, I a^2 \frac{\partial B_x}{\partial x}.$$

More generally :

$$\vec{F} = (\vec{\mu} \cdot \vec{\nabla}) \vec{B}$$

The same as electric dipoles.

~~q~~ So if the magnetic moment changes by $\Delta\mu$

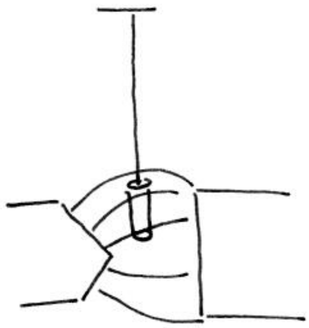
$$\Delta\vec{F} = (\Delta\vec{\mu}) \cdot \nabla \vec{B}$$

$$= -\left(\frac{q^2 r^2}{4m}\right)(B \cdot \nabla) B$$

$\Rightarrow$ The magnetic material is repelled; moves towards weaker magnetic fields!

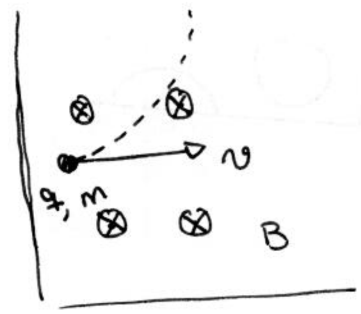
To summarize : ~~magnetic~~ in the presence of an inhomogeneous magnetic field, ~~magnetic~~ matter should move towards weaker magnetic field. This is a consquence of Lenz's law. This is ~~so~~ what our classical theory tells us.

In reality:

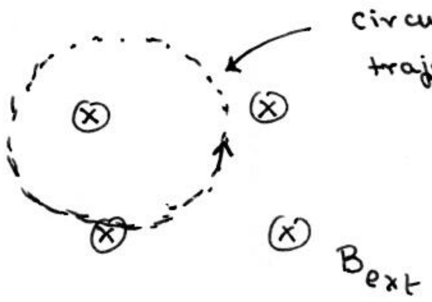


some materials move slightly towards weaker magnetic field	Diamagnetic material (Bismuth, Oxygen ...)	weak
some materials move towards stronger magnetic field	Paramagnetic material (e.g. Aluminium ...)	moderate
Some materials move hugely towards stronger magnetic field	Ferromagnetic materials (Iron, Nickel, certain alloys...)	very strong.

14·3 Classical physics gives neither diamagnetism nor paramagnetism.

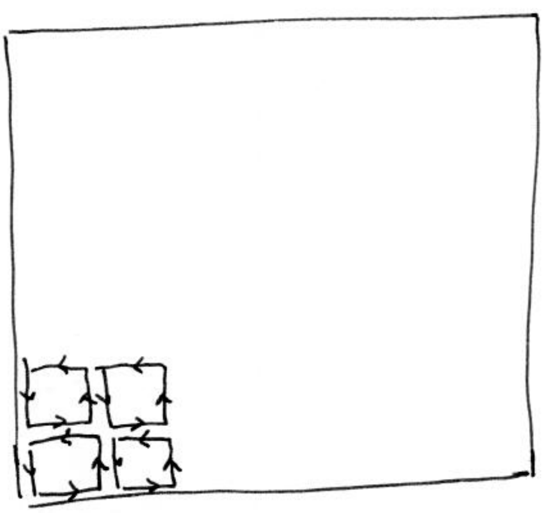


$$R = \frac{mv}{qB}$$



$\odot \, \mu$ ⟶ Induced dipole moment of the loop that opposes B_{ext}.

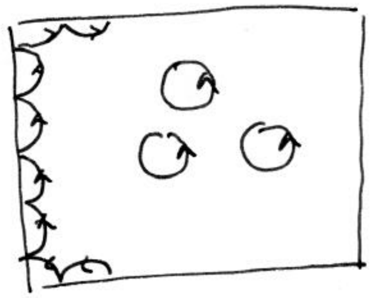
Now put the system in a closed box



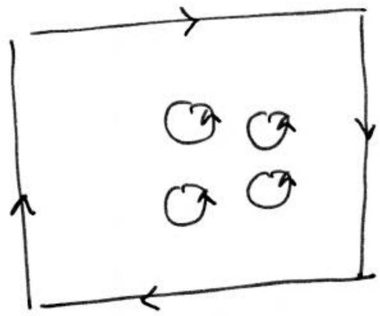
And all the induced moments would add up, right?

Actually wrong.

There would be lots of incomplete orbits.



which gives the opposite sign of current in the boundary



The magnetic moment of the boundary currents will cancel the magnetic moment of the internal currents. So, in a closed system, ~~(a~~ ~~syst~~ according to classical physics, the induced dipole moment $\Delta\mu = 0$

There exists ~~not~~ neither diamagnetism, nor paramagnetism!

14.4 What actually happens?

$$\mu = -g\left(\frac{q_e}{2m}\right) J \qquad -q_e : \text{electronic charge.}$$

Landé g factor

$$g = 1 \quad \text{for orbital } J$$
$$\;\;= 2 \quad \text{for spin.}$$

The electron spin has often been thought of like the angular momentum of earth about it's own axis. But, in principle, that is wrong because to generate the observed angular momentum the speed at which the surface of electron would have to move is too high. Spin should rather be taken as a fundamental property of elementary particles, like their mass and charge.

The energy of a dipole in an external field:

$$U_{mag} = -\vec{\mu}\cdot\vec{B}$$

$$= -\mu_z B \qquad \text{for } \vec{B} \text{ along } z.$$

$$= +\,g\left(\frac{q_e}{2m}\right) J_z B$$

The quantum mechanics also says that

$$J_z = j\hbar, (j-1)\hbar, \ldots, -j\hbar$$

where $\hbar = \frac{h}{2}$, h = Plack's constant

only discrete values of J_z are allowed.

$$U_{mag} = g\mu_B B \frac{J_z}{\hbar}, \quad \mu_B = \frac{q_e \hbar}{2m}$$

Bohr magneton.

$$(J_z/\hbar) \rightarrow j, (j-1), \ldots, -j$$

~~So, when a magnetic field is turned on~~

In the absence of any external magnetic field all the different angular momentum states are equally probable and then on average the magnetic moment of the system is zero. ~~If there is an external magnetic field then~~ This is like the case of molecules with no permanent electric dipole moments. But there are also molecules with permanent magnetic dipole moments, e.g., chromium, iron, nickel, magnesium.

Either way, in the absence of a quantum mechanical insight, we just ~~take~~ assume that materials can develop an average magnetization

$$M = \underset{\substack{\uparrow \\ \text{number of} \\ \text{particles per unit vol.}}}{N} \langle \vec{\mu} \rangle_{av}$$

and

$$M = \underset{\substack{\uparrow \\ \text{magnetic susceptibily}}}{\chi_M} B$$

~~classical~~
Depending on whether this ~~susceptibily~~ susceptibility is positive or negative we can have ~~diam~~ paramagnetic or diamagnetic material..

~~Following the same route as dielectrics we can write~~

$$\cancel{\boxed{\nabla \cdot M = - \rho_M}} \leftarrow \text{purely mathematical}$$

Just as the macroscopic electric field in matter was thought of as due to free and polarized charges, we can attribute macroscopic magnetic field to free current (conduction current, the current that we can turn off and on) and bound current (molecular currents)

$$J = J_{cond} + J_{pol} + J_{mag}$$

from the motion of ρ_{pol}

Other "mysterious" molecular currents

From Maxwell:

$$\nabla \times B = \mu_0 J + \epsilon_0 \mu_0 \frac{\partial E}{\partial t}$$

$$= \mu_0 \left(J_{cond} + J_{pol} + J_{mag} \right) + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

And then we attribute

$$J_{mag} = \nabla \times M$$

magnetisation, average of molecular dipole moments.

Note here that M could be due to spin, in which case associating a real physical molecular current as it's source is impossible. This relation should be thought of as a macroscopic relation relating physical measurable magnetization to a set of ~~to~~ ~~mathe~~ mathematical bound currents.

$$\nabla \times B = \mu_0 J_{cond} + \mu_0 J_{pol} + \mu_0 \nabla \times M + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\Rightarrow \quad \nabla \times (B - \mu_0 M) = \mu_0 J_{cond} + \mu_0 J_{pol} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Note that $\quad \partial_t \rho_{pol} + \nabla \cdot J_{pol} = 0$

conservation of polarized charge.

Also $\quad \nabla \cdot P = - \rho_{pol}$

$$\Rightarrow \quad - \nabla \cdot \partial_t P + \nabla \cdot J_{pol} = 0$$

$$\Rightarrow \quad J_{pol} = + \partial_t P$$

Substituting back:

$$\nabla \times \left(B - \mu_0 M \right) = \mu_0 J_{cond} + \mu_0 \partial_t P + \mu_0 \epsilon_0 \partial_t E$$

$$\Rightarrow \quad \nabla \times \left(B - \mu_0 M \right) = \mu_0 J_{cond} + \mu_0 \epsilon_0 \partial_t \left(E + P/\epsilon_0 \right)$$

Remember from last lecture that

$$\nabla \cdot E = \left(\rho_f + \rho_p \right) \frac{1}{\epsilon_0}$$

$$= \frac{1}{\epsilon_0} \rho_f - \frac{1}{\epsilon_0} \nabla \cdot P$$

$$\Rightarrow \quad \nabla \cdot \left(E + \frac{P}{\epsilon_0} \right) = \rho_f / \epsilon_0$$

$$\Rightarrow \quad \nabla \cdot \underbrace{\left(E + P/\epsilon_0 \right)}_{\epsilon_0 D} = \rho_f \,, \qquad \nabla \cdot D = \rho_f$$

$$\Rightarrow \quad \nabla \times \left(B - \mu_0 M \right) = \mu_0 J_{cond} + \mu_0 \frac{\epsilon_0}{\epsilon_0} \frac{\partial D}{\partial t}$$

Define $\quad H = B - \mu_0 M$

$$\boxed{\nabla \times H = \mu_0 J_{cond} + \mu_0 \frac{\epsilon_0}{\epsilon_0} \frac{\partial D}{\partial t}}$$

Maxwell's eqn in presence of matter.

Some simplification :

$$\nabla \times H = \mu_0 J_f + \frac{\mu_0 \epsilon_0}{\epsilon_0} \frac{\partial D}{\partial t}$$

using $c^2 = \frac{1}{\mu_0 \epsilon_0}$

$$c^2 \nabla \times H = \frac{1}{\epsilon_0} J_f + \frac{1}{\epsilon_0} \frac{\partial D}{\partial t}$$

$$\Rightarrow \boxed{\epsilon_0 c^2 \nabla \times H = J_f + \frac{\partial D}{\partial t}}$$

To summarize :

In vacuum

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0$$

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$c^2 = \frac{1}{\mu_0 \epsilon_0}$$

In material

$$\nabla \cdot D = \rho_f$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0$$

$$\epsilon_0 c^2 \nabla \times H = J_f + \frac{\partial D}{\partial t}$$

The material side is not complete but can be made complete by adding

$$\left.\begin{aligned} D &= \epsilon E \\ H &= \mu B \end{aligned}\right\} \text{ linear materials}$$

Then we have:

$$\nabla \cdot D = \rho_f$$

$$\epsilon \mu \, \nabla \times D = -\frac{\partial H}{\partial t} \qquad (\mu, \epsilon \text{ constant in space})$$

$$\nabla \cdot H = 0$$

$$\epsilon_0 c^2 \, \nabla \times H = J_f + \frac{\partial D}{\partial t}$$

In a space free of "free" charges and currents

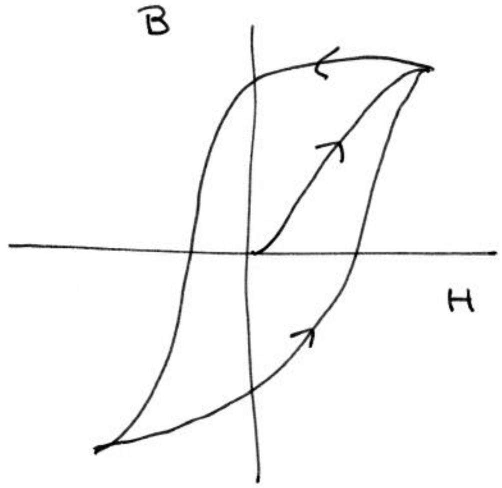
$$\nabla \cdot D = 0, \quad \nabla \cdot H = 0$$

$$\epsilon \mu \, \nabla \times D = -\frac{\partial H}{\partial t}, \quad \epsilon_0 c^2 \, \nabla \times H = \frac{\partial D}{\partial t}$$

$$\Rightarrow \frac{\partial^2 H}{\partial t^2} = -\epsilon \mu \, \nabla \times \frac{\partial D}{\partial t}$$

$$= -\epsilon \mu \, \nabla \times \nabla \times \epsilon_0 c^2 H$$

In ~~me~~ ferromagnetic materials μ is not a constant, but even depends on the history of the material



Hysterisis curve