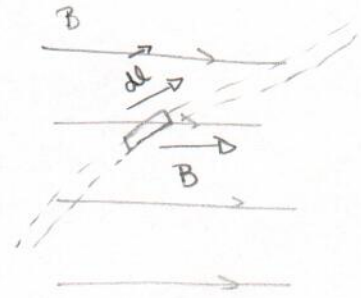
Lecture 6 — Electromagnetic induction.

6.1

Force and torque on a current loop

Let us start by what we already know. If a current carrying wire is placed in a magnetic field the magnetic forces can act on the moving charges in the wire. This in turn can move the wire or rotate a current loop. Thus, we now have a way of generating mechanical force from an electromagnetic one.



The force on a current carrying wire:

charge

$$d\vec{F} = n q \, (\vec{v} \times \vec{B}) \, (dl \cdot A)$$

cross-sectional area.

volume density of charge careers

assuming all the charges move with same velocity

$$= I \, d\vec{l} \times \vec{B}$$

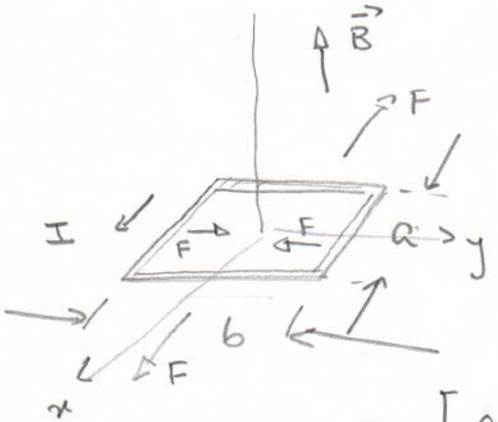
where $I = A n q v$

The force on a current loop is then

$$\vec{F} = I \oint_{\text{loop}} d\vec{l} \times \vec{B}$$

Example 6·1

Force on a square loop:



Total force

$$\vec{F} = I\oint d\vec{l} \times \vec{B}$$

$$= I\left[a\,\hat{x} \times \vec{B} + b\,\hat{y} \times \vec{B} - a\,\hat{x} \times \vec{B} - b\,\hat{y} \times \vec{B}\right]$$

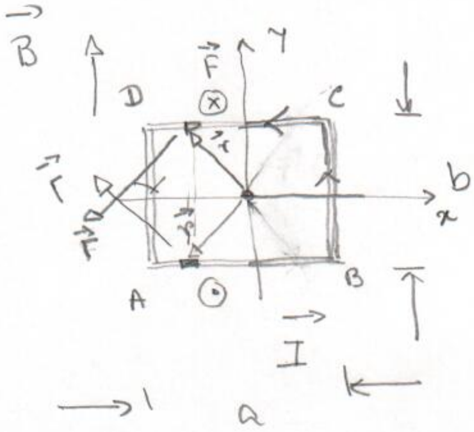
$= 0$ for any planar loop, because if $\vec{B}$ is constant and perpendicular to the plane,

$$\vec{F} = I\left(\oint_{\Gamma} d\vec{l}\right) \times B = 0$$

beauese $\oint d\vec{l} = 0$.

Example 6.2

Now consider a different loop



$$\vec{F} = I \oint d\vec{l} \times \vec{B}$$

$$= 0$$

The force must still be zero as $\vec{B}$ is a constant but what about the torque?

$\hat{x}$

$\hat{z}$ $\hat{y}$

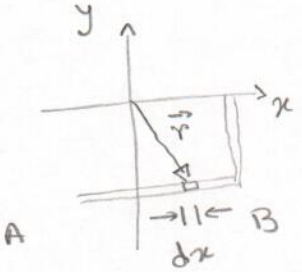
The torque

$$\vec{\Gamma} = \oint \vec{r} \times d\vec{F}$$

$$= I \oint \vec{r} \times d\vec{l} \times \vec{B}$$

$$= I \left[\int_A^B \vec{r} \times dx\,\hat{x} \times \hat{y}\, B_0 + \int_C^D \cdot \right]$$

$$= I B_0 \left[\int_A^B \vec{r} \times (\hat{z})\, dx - \int_C^D (\vec{r} \times \hat{z})\, dx \right]$$



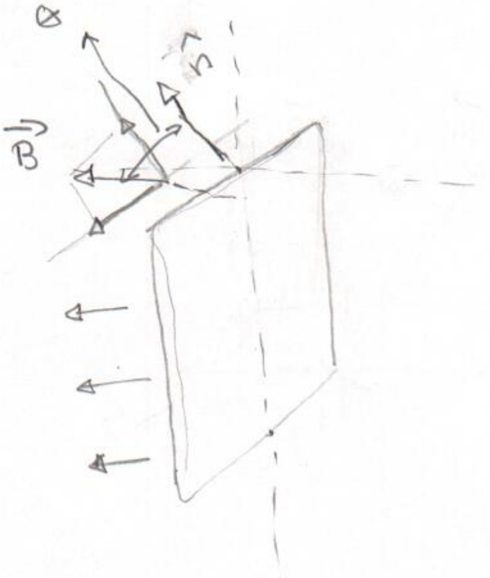
over $A \to B$ $\quad \vec{r} = \left(-\frac{b}{2}\right)\hat{y} + x\,\hat{x}$

over $C \to D$ $\quad \vec{r} = \frac{b}{2}\hat{y} + x\,\hat{x}$

$$\vec{\Gamma} = I B_0 \int_{-\frac{a}{2}}^{+\frac{a}{2}} \left(-\frac{b}{2}\right)\hat{x}\, dx + (-\hat{y}) \int_{-a/2}^{+a/2} x\, dx$$

$$+ \frac{b}{2} \int_{-a/2}^{+a/2} \hat{x}\, dx + \hat{y} \int_{-a/2}^{+a/2} x\, dx$$

$$= \hat{x}\, I B_a\, ab$$



If the loop made an angle with the magnetic field ?

Then only the component $\vec{B}$ in the plane of the loop would matter.

$$\vec{\Gamma} = I\, a b\, B \sin\theta$$
$$= I S\, \hat{n} \times \vec{B}$$

The torque is zero when the loop is perpendicular to $\vec{B}$.

Remember that the magnetic dipole moment of the loop is $\vec{m} = I S \hat{n}$

$$\Rightarrow \quad \boxed{\vec{\Gamma} = \vec{m} \times \vec{B}}$$

6·2 Motors and galvanometers.

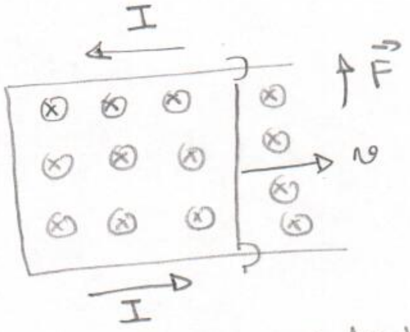
The torque can be used to raise a weight or do work of any other sort. Thus we have now a way to convert electric energy to mechanical energy: a motor.

The torque can be increased by putting many turns of the current on the same mechanical loop.

Mounted on a quartz fibre the same torque can be ballanced against a mechanical torque. Once callibrated, the instrument can be used to detect very small amount of curren; this is a galvanometer.

So far we have not introduced any new physics but merely the Lorentz force.

Now consider the following example.



I move a conductor in the presence of a magnetic field. A Lorentz force will set up a current. So moving a conductor in a magnetic field I can convert mechanical energy to electrical energy.

I can also send signals along the wire. This is the beginning of telegraph.

Clearly the magnetic field is set up by another set of currents. What happens, if instead of moving the bar, I change the current that set up the magnetic field in the first place?

This question was asked by Faraday.

And the answer is given by the flux rule.

- Experiment I

  move a magnet near a coil and try to detect the current in the coil by a galvanometer.

- Experiment II : A magnet dropping under gravity through a conducting ring.

- Experiment III : The jumping ring.

6·3 Laws of induction.

Rate of change of magnetic flux through a circuit sets up an electromotive force (EMF), that is given by

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

where $\Phi = \int \vec{B} \cdot \hat{n}\, ds$

The flux can change because:

(a) we move something mechanically

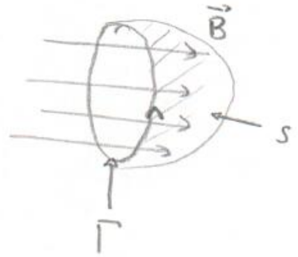
(b) The magnetic field changes

The flux rule has exactly the same form in both the cases.

Now consider a circuit that is fixed but the magnetic field is changing. The change in flux sets up an EMF. This is the sum of all the electrical forces on the charges. In other words we could identify

$$\mathcal{E} = \oint_{\text{circuit}} \vec{E} \cdot d\vec{l}$$

$$\Rightarrow \quad \oint_{\Gamma} \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_{S} \vec{B} \cdot \hat{n}\, ds$$



$$\Rightarrow \quad \int_{S} \left(\vec{\nabla} \times \vec{E}\right) \cdot \hat{n}\, ds = -\int_{S} \frac{\partial \vec{B}}{\partial t} \cdot \hat{n}\, ds$$

for any <u>suface S</u>

$$\Rightarrow \quad \boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

<u>Comment</u>

1. There are "exceptions" to the flux rule where a motion sets up and EMF although the flux remains constant. An example is the Faraday disk dynamo. In general, the best way to analyze a problem of electromagnetic induction is to use,

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

and
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

2. The negative sign in the law of induction shows that the EMF is set up such that it opposes the change of flux. This is called the Lenz's law.